Curve	Gradient at (0, 1)
$y = 2^x$	0.6931
$y = 3^x$	1.0986
$y = 4^x$	1.3863

This suggests there is a curve in the family $y = a^x$ whose gradient is exactly 1 at the point (0, 1). This is the definition of the curve $y = e^x$. In fact, the gradient of the curve $y = e^x$ is equal to the value of y at every point on the curve.

$y = e^x$ is the curve whose gradient equals its y value at every point.

The value of e is just over 2.71. Like π, it is an irrational number, which means it cannot be written exactly as a fraction.

Note: you may also see the notation $y = \exp(x)$.

Transformations

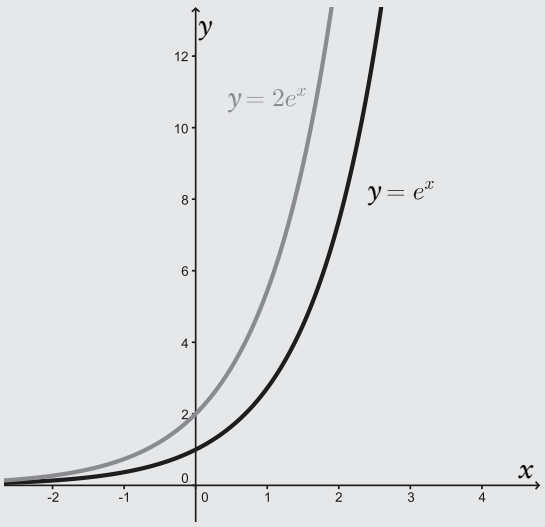
You will need to perform transformations on the exponential curve $y = e^x$. You will use the usual rules:

- $y = -f(x)$ Reflection in x-axis
- $y = af(x)$ Stretch in y-direction, scale factor a
- $y = f(x) - a$ Translation by a units in negative y-direction
- $y = f(ax)$ Stretch in x-direction, scale factor $\frac{1}{a}$. (Shrink, scale factor a)
- $y = f(x - a)$ Translation by a units in positive x-direction
- $y = f(-x)$ Reflection in y-axis

EXAMPLE 1

Sketch the curves $y = e^x$ and $y = 2e^x$ on the same diagram.

The curve $y = 2e^x$ is a stretch of the curve $y = e^x$ parallel to the y-axis.
It passes through the point (0, 2).



EXAMPLE 2

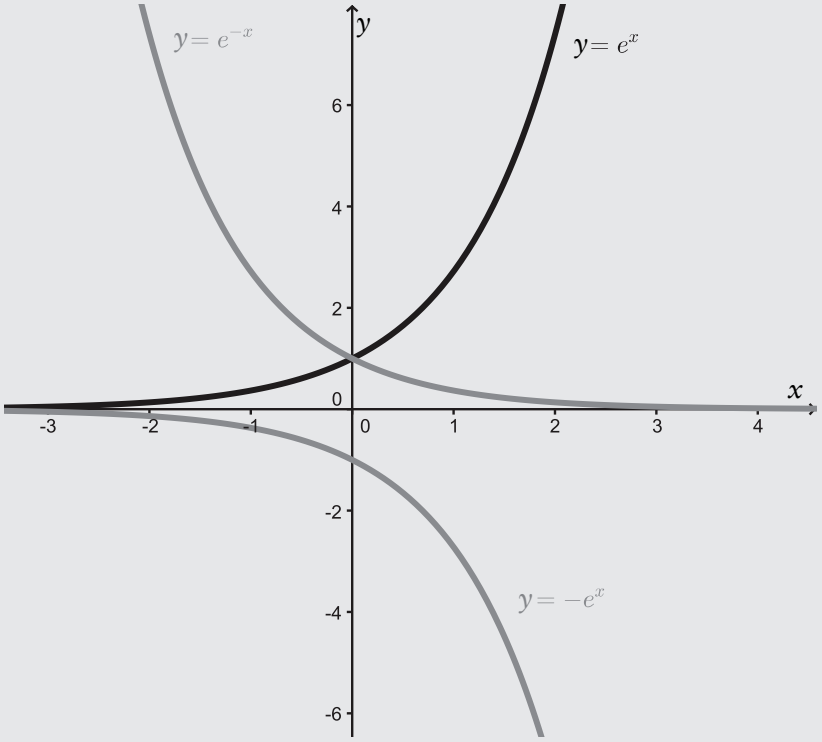
Sketch the three curves $y = e^x$, $y = e^{-x}$ and $y = -e^x$ on the same diagram.

$y = e^{-x}$ is a transformation of the form $y = f(-x)$.

It represents a reflection of $y = e^x$ in the y-axis.

$y = -e^x$ is a transformation of the form $y = -f(x)$.

It represents a reflection of $y = e^x$ in the x-axis.



EXERCISE

EXERCISE 5B

1. Sketch each curve and $y = e^x$ on the same diagram. In each case, state which transformation(s) could transform the graph of $y = e^x$ into the graph of the equation given.

   a) $y = e^{2x}$ b) $y = e^{-2x}$ c) $y = -e^{-2x}$ d) $y = e^{x+1}$ e) $y = 3e^{2x}$

   f) $y = e^{x-1}$ g) $y = 1 - e^x$ h) $y = -3e^x$ i) $y = -3e^{-x}$ j) $y = e^{x/2}$

   k) $y = \frac{e^x}{2}$ l) $y = \frac{e^x}{2} - 1$ m) $y = 2 - 3e^x$ n) $y = 2 - e^{x/2}$ o) $y = e^x - 1$

2. The curve $y = e^x$ has a gradient of 1 at the point (0, 1). By referring to your sketches in question 1, or otherwise, state the gradient of the following curves at the point where the curve crosses the y-axis.

   a) $y = 1 - e^x$ b) $y = -3e^x$ c) $y = -3e^{-x}$ d) $y = \frac{e^x}{2}$ e) $y = \frac{e^x}{2} - 1$

   f) $y = 2 - e^{x/2}$ g) $y = e^x - 1$

EXERCISE